Threshold Resummation for $W^{\pm}$ Production at RHIC

Asmita Mukherjee
Indian Institute of Technology, Mumbai, India

- Threshold logarithms
- Resummation of threshold logs
- Inverse transformation and matching
- Numerical results for RHIC

In collaboration with Werner Vogelsang (BNL)

AM and W. Vogelsang PRD 73, 074005 (2006)

Objective

- Process to look at : $\vec{p}p \longrightarrow W^{\pm}X$

- W mass sets a large scale : cross section factorizes; LO $q\bar{q}' \longrightarrow W^{\pm}$

For W^{+} : dominant contribution from $u\bar{d} \longrightarrow W^{+}$

- Single spin asymmetry due to parity violation at LO

$$A_L = \frac{\Delta u(x_1^0, M_W^2)\bar{d}(x_2^0, M_W^2) - \Delta\bar{d}(x_1^0, M_W^2)u(x_2^0, M_W^2)}{u(x_1^0, M_W^2)\bar{d}(x_2^0, M_W^2) + \bar{d}(x_1^0, M_W^2)u(x_2^0, M_W^2)};$$

pdfs probed at scale M_W ; $x_{1,2}^0 = \frac{M_W}{\sqrt{S}}\, e^{\pm\eta}$

$\sqrt{S}$ is the CM energy of the hadrons and η is the rapidity of the W

- Provides access to $\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}$ at RHIC.

- Note : in inclusive DIS via photon exchange only the combination $\Delta q + \Delta\bar{q}$ can be accessed

- Other possibility : semi inclusive DIS : sensitive to $\Delta q(x)\, D_q^h(z) + \Delta\bar{q}(x)\, D_{\bar{q}}^h(z)$, for a given quark flavor

Threshold Logarithms

- NLO : $q\bar{q}' \to W^\pm g$ and $qg \to W^\pm q'$ contribute

- Corrections near 'partonic threshold' : when the initial partons have just enough energy to produce $W^\pm$: $z = M_W^2/\hat{s} \to 1$

- Phase space for real gluon emission vanishes; virtual corrections fully allowed

- Cancellation of singularities between real and virtual diagrams give large logarithmic 'Sudakov' corrections to $q\bar{q}$ cross section

- Interplay of the steeply falling pdfs with the partonic cross sections : Partonic threshold can make a significant contribution to the cross section even if the hadronic process is relatively far from threshold i.e. $\frac{M_W^2}{S} << 1$; RHIC : not far from threshold

- If $M_W^2/S \sim 1$, threshold region completely dominates the cross section ($\sqrt{S} = 200$ GeV at RHIC

- Sufficiently close to threshold : perturbative series is only useful if large logs is taken into account to all orders in α_s : resummation.

Threshold Resummation for Drell-Yan

- Threshold resummation first derived for Drell-Yan

G. Sterman, Nucl. Phys. B **281**, 310 (1987)
S. Catani and L. Trentadue, Nucl. Phys. B **327**, 323 (1989); Nucl. Phys. B **353**, 183 (1991).

- Most important logs (leading logs LL) are of the form $\alpha_s^k[\ln^{2k-1}(1-z)/(1-z)]_+$ at the kth order in α_s, where $z = \frac{M_W^2}{\hat{s}}$

- Threshold resummation in D-Y known to NNLL; $\alpha_s(M_W)$ sufficiently small and NLL is sufficient for our purpose here

- Rapidity distribution of W is of interest in RHIC

- Here we perform threshold resummation of the rapidity distribution of the W production cross section

- Threshold resummation of Drell-Yan production at fixed rapidity briefly discussed in

E. Laenen and G. Sterman, FERMILAB-CONF-92-359-T

- For prompt photon production, threshold resummation for rapidity distribution has been done in

G. Sterman and W. Vogelsang, JHEP **0102**, 016 (2001)

Cross Section for W Production

$$
\begin{aligned}
\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}\eta} \quad &= \quad \mathcal{N} \sum_{i,j} c_{ij} \int_{x_1^0}^{1} \mathrm{d}x_1 \int_{x_2^0}^{1} \mathrm{d}x_2 \\
&\times \Bigg\{ D_{q\bar{q}} \left(x_1, x_2, x_1^0, x_2^0, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2} \right) \Big[-\Delta q_i(x_1,\mu^2) \bar{q}_j(x_2,\mu^2) + \Delta\bar{q}_i(x_1,\mu^2) q_j(x_2,\mu^2) \Big] \\
&\quad + \Delta D_{gq} \left(x_1, x_2, x_1^0, x_2^0, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2} \right) \Delta G(x_1,\mu^2) \left[q_j(x_2,\mu^2) - \bar{q}_j(x_2,\mu^2) \right] \\
&\quad + D_{qg} \left(x_1, x_2, x_1^0, x_2^0, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2} \right) \left[-\Delta q_i(x_1,\mu^2) + \Delta\bar{q}_i(x_1,\mu^2) \right] G(x_2,\mu^2) \Bigg\}
\end{aligned}
$$

$\mathcal{N} = \frac{\pi G_F M_W^2 \sqrt{2}}{3S}$, c_{ij} are the coupling factors for $W^\pm$ bosons; $\mu \sim M_W$ denotes the renormalization/factorization scales

$$
(\Delta) D_{ij} = (\Delta) D_{ij}^{(0)} + \frac{\alpha_s(\mu^2)}{2\pi} (\Delta) D_{ij}^{(1)}
$$

T. Gehrmann, Nucl. Phys. B **534**, 21 (1998)

Cross Section for W Production (Contd.)

- $D_{q\bar{q}}$, ΔD_{gq}, D_{qg} hard-scattering functions

- Helicity conservation in QCD and the $V-A$ structure of the $Wq\bar{q}'$ vertex : when the polarized parton is a quark, spin-dependent partonic cross section is the negative of the unpolarized one $\rightarrow$ omitted Δ

- Each of the hard-scattering functions D_{ij} (or ΔD_{ij}) is a perturbative series in the strong coupling $\alpha_s(\mu^2)$:

$$
\begin{aligned}
D_{ij}\left(x_1, x_2, x_1^0, x_2^0, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2}\right) \quad &= \quad D_{ij}^{(0)}\left(x_1, x_2, x_1^0, x_2^0\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} D_{ij}^{(1)}\left(x_1, x_2, x_1^0, x_2^0, \frac{M_W^2}{\mu^2}\right) + \ldots \ .
\end{aligned}
$$

Cross Section for W Production (Contd.)

Only the $q\bar{q}'$ process has a lowest-order [$\mathcal{O}(\alpha_s^0)$] contribution

$$\begin{aligned} D^{(0)}_{q\bar{q}}(x_1,x_2,x_1^0,x_2^0) &= \delta(x_1-x_1^0)\,\delta(x_2-x_2^0)\;, \\ \Delta D^{(0)}_{gq}(x_1,x_2,x_1^0,x_2^0) &= D_{qg}(x_1,x_2,x_1^0,x_2^0)\;=\;0\;. \end{aligned}$$

• Hard coefficients contain distributions in $(x_1-x_1^0)$ and $(x_2-x_2^0)$: addressed by threshold resummation

• Products of two ‘plus”-distributions, or a product of a ‘plus”-distribution and a delta-function are leading near threshold

• Come only in the coefficient $D_{q\bar{q}}$

• Resummation at fixed, arbitrary rapidity

Threshold Resummation in Mellin Space

Define a double moment of the cross section

$$\Delta\tilde{\sigma}(N,M) \quad \equiv \quad \int_0^1 d\tau\, \tau^{N-1} \int_{-\ln\frac{1}{\sqrt{\tau}}}^{\ln\frac{1}{\sqrt{\tau}}} d\eta\, \mathrm{e}^{iM\eta}\, \frac{d\Delta\sigma}{d\eta}$$

We introduce parton level variables $z = \tau/x_1 x_2$ and $\hat{\eta} = \eta - \frac{1}{2}\ln(x_1/x_2)$; $\tau = \frac{M_W^2}{S}$

Moments are defined as

$$f^N(\mu^2) \equiv \int_0^1 dx\, x^{N-1} f(x,\mu^2),$$

$$\Delta\tilde{\mathcal{D}}_{ij}(N,M) \equiv \int_0^1 dz\, z^{N-1} \int_{-\ln\frac{1}{\sqrt{z}}}^{\ln\frac{1}{\sqrt{z}}} d\hat{\eta}\, \mathrm{e}^{iM\hat{\eta}}\, \left(x_1 x_2 \Delta\mathcal{D}_{ij}\right)\ ,$$

We then have

$$\Delta\tilde{\sigma}(N,M) = \mathcal{N}\,\Delta f_i^{N+iM/2}\, f_j^{N-iM/2}\, \Delta\tilde{\mathcal{D}}_{ij}(N,M)\ .$$

Threshold Resummation (contd.)

We have, near threshold

$$\int_0^1 dz\, z^{N-1} \int_{-\ln\frac{1}{\sqrt{z}}}^{\ln\frac{1}{\sqrt{z}}} d\hat{\eta}\, \mathrm{e}^{iM\hat{\eta}}\, x_1 x_2 D_{q\bar{q}} = \int_0^1 dz\, z^{N-1}\, 2\cos\left(M\ln\frac{1}{\sqrt{z}}\right)\omega_{q\bar{q}}$$

where

$$\begin{aligned}\omega_{q\bar{q}} \quad &= \quad \delta(1-z) + \frac{\alpha_s}{2\pi} C_F \Bigg[8\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{4}{(1-z)}_+ \ln\frac{M_W^2}{\mu^2} \\ &\quad + \left(-8 + \frac{2\pi^2}{3} + 3\ln\frac{M_W^2}{\mu^2}\right)\delta(1-z)\Bigg] + O(1-z)\ ,\end{aligned}$$

Cos term can be expanded as

$$\cos\left(M\ln\frac{1}{\sqrt{z}}\right) = 1 - \frac{(1-z)^2 M^2}{8} + \mathcal{O}\left((1-z)^4 M^4\right)\ .$$

Can be approximated by 1 as z is very close to 1

Threshold Resummation (contd.)

Mellin moments of NLO terms become

$$\int_0^1 dz\, z^{N-1}\, C_F \left[8 \left(\frac{\ln(1-z)}{1-z} \right)_+ + \frac{4}{(1-z)_+} \ln \frac{M_W^2}{\mu^2} + \left(-8 + \frac{2\pi^2}{3} + 3 \ln \frac{M_W^2}{\mu^2} \right) \delta(1-z) \right]$$

$$= \quad 2C_F \left[2\ln^2(\bar{N}) + \left(\frac{3}{2} - 2\ln(\bar{N}) \right) \ln \frac{M_W^2}{\mu^2} - 4 + \frac{2\pi^2}{3} \right] + \mathcal{O}\left(\frac{1}{N} \right) \,,$$

Here

$$\bar{N} = N \mathrm{e}^{\gamma_E} \,.$$

- Independent of M upto small corrections suppressed near threshold
- The N-dependence is identical to that of the rapidity-integrated cross section
- Dependence on rapidity is then entirely contained in the pdfs, their moments are shifted

by M-dependent terms

Resummed Cross Section

To NLL accuracy, one finds

$$\int_0^1 dz\, z^{N-1} \int_{-\ln\frac{1}{\sqrt{z}}}^{\ln\frac{1}{\sqrt{z}}} d\hat\eta\, \mathrm{e}^{iM\hat\eta}\, x_1 x_2 D^{\rm res}_{q\bar q}\left(x_1, x_2, x_1^0, x_2^0, \alpha_s(\mu^2), \frac{M_W^2}{\mu^2}\right)$$

$$= \quad \exp\left\{C_q\left(\alpha_s(\mu^2), \frac{M_W^2}{\mu^2}\right) + 2\ln\bar N\, h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda, \frac{M_W^2}{\mu^2}\right)\right\} \;,$$

where $\lambda = b_0\alpha_s(\mu^2)\ln\bar N$, b_0 is constant

Contributions from quark-gluon channels down by $1/N$: will be included at NLO level

C_q collects mostly hard virtual corrections

$h^{(1)}$ contains LL and $h^{(2)}$ contains NLL terms

Resummation in Mellin space, inverted and matched with NLO

Used 'minimal' expansion of the resummed exponent

S. Catani, M. L. Mangano, P. Nason and L. Trentadue, Nucl. Phys. B **478**, 273 (1996)

Threshold resummation

Improve the formula by taking subleading terms :

Resummed exponent contains a term which is the LL expansion of

$$\int_{\mu^2}^{M_W^2/\bar{N}^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(k_T^2)}{\pi} \left[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}\right] .$$

$\left[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}\right]$: comes from the large-N limit of the moments of the LO splitting function ΔP_{qq}^N

Modify the resummation by replacing

$$\left[-2A_q^{(1)} \ln \bar{N} - B_q^{(1)}\right] \longrightarrow C_F \left[\frac{3}{2} - 2S_1(N) + \frac{1}{N(N+1)}\right]$$

RHS: moments of the full LO non-singlet splitting function
Includes also the $\sim \ln(\bar{N})/N$ terms in NLO $q\bar{q}$, cross sections

Inverse Transformation and matching

- Final step : take the Mellin and Fourier inverse transforms of $\Delta\tilde{\sigma}(N, M)$

$$\frac{d\Delta\sigma^{\rm res}}{d\eta} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dM\, {\rm e}^{-iM\eta}\, \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\, \tau^{-N}\, \Delta\tilde{\sigma}^{\rm res}(N, M)$$

- Choose the contour of integration

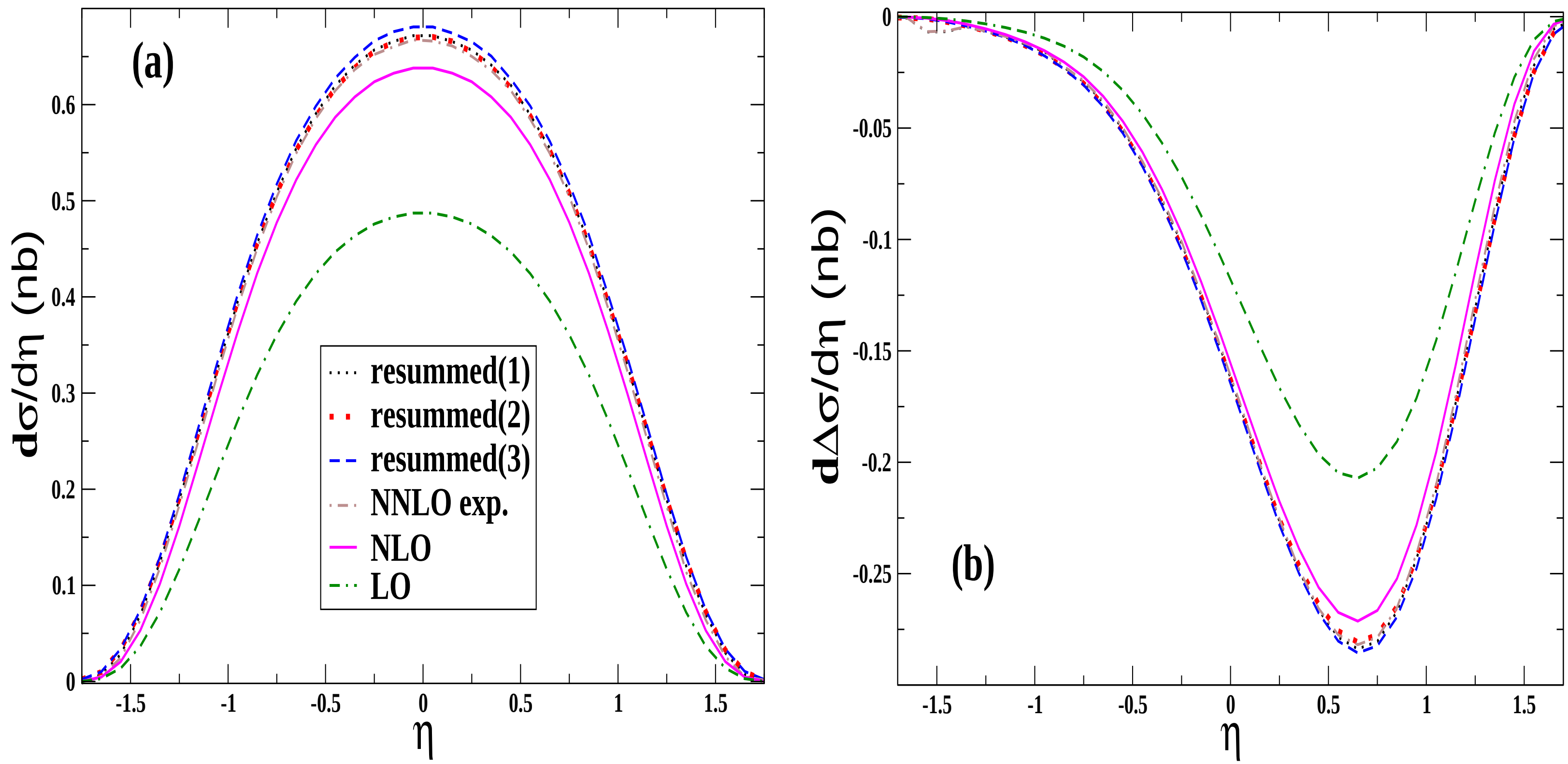
G. Sterman and W. Vogelsang, JHEP **0102**, 016 (2001)

- To keep the full information contained in the NLO calculation, perform a ‘matching” of the NLL resummed cross section to the NLO :
Subtract from the resummed expression its $\mathcal{O}(\alpha_s)$ expansion,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dM\, {\rm e}^{-iM\eta}\, \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN\, \tau^{-N} \left[\Delta\tilde{\sigma}^{\rm res}(N, M) - \Delta\tilde{\sigma}^{\rm res}(N, M)\Big|_{\mathcal{O}(\alpha_s)}\right] ,$$

- Then add full NLO cross section.

W^+ Production at RHIC ($\sqrt{S} = 500$ GeV)



Resummed(1) : threshold-resummed cross section without subleading $\ln(\bar{N})/N$ terms and without the $\cos\left(M \ln \frac{1}{\sqrt{z}}\right)$ term

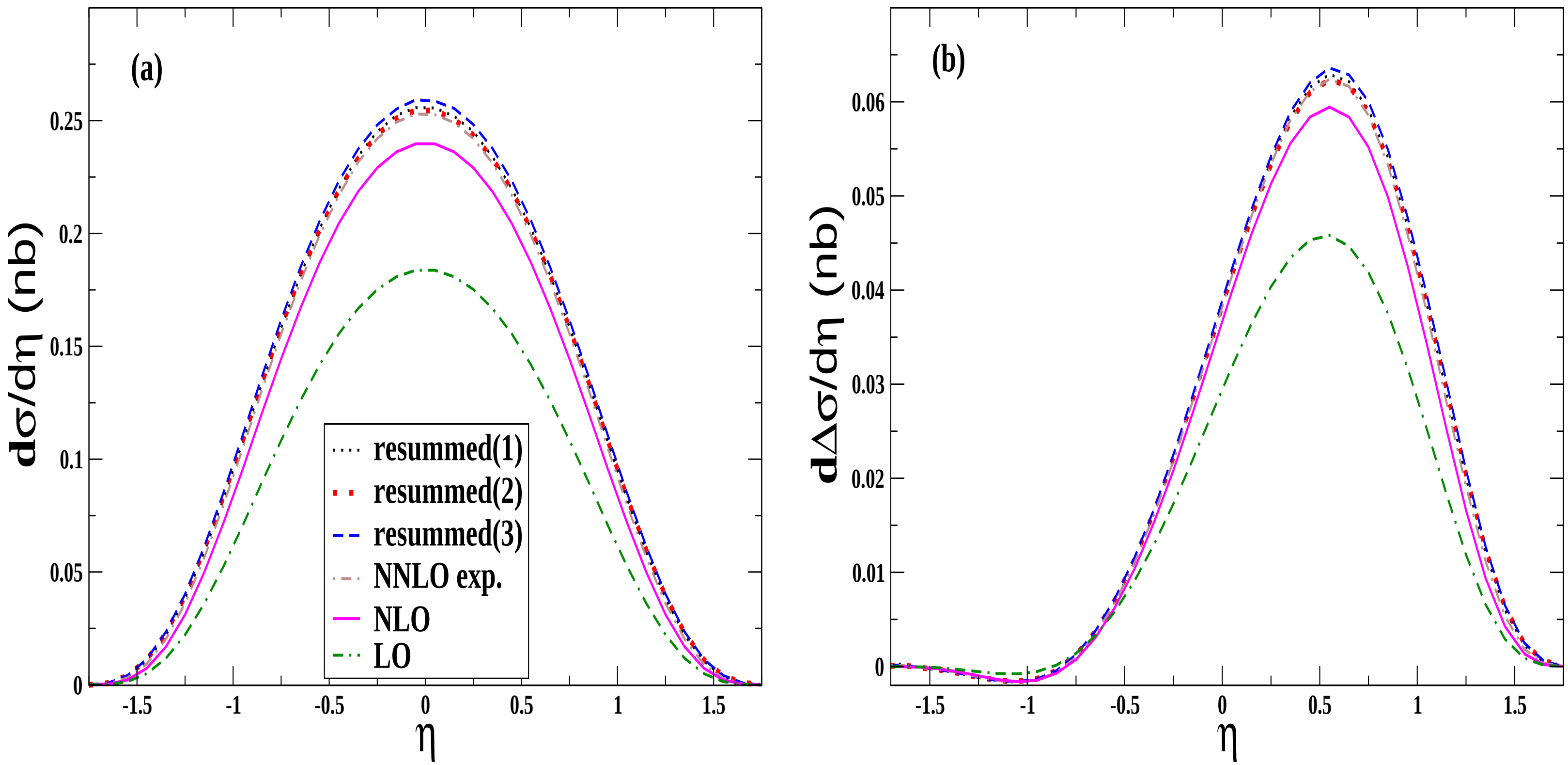
Resummed(2) : the Cosine term is included

Resummed(3): with $\ln(\bar{N})/N$ terms

NNLO exp : NNLO expansion of the 'resummed(1)' result

pdfs used : NLO GRSV standard set, NLO GRV

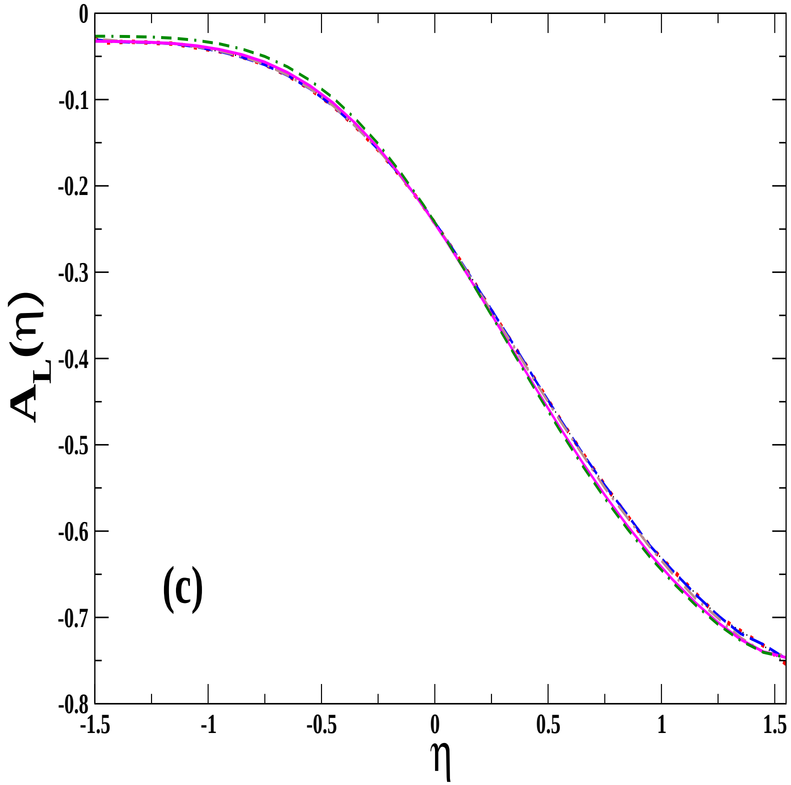
W^- Production at RHIC ($\sqrt{S} = 500$ GeV)

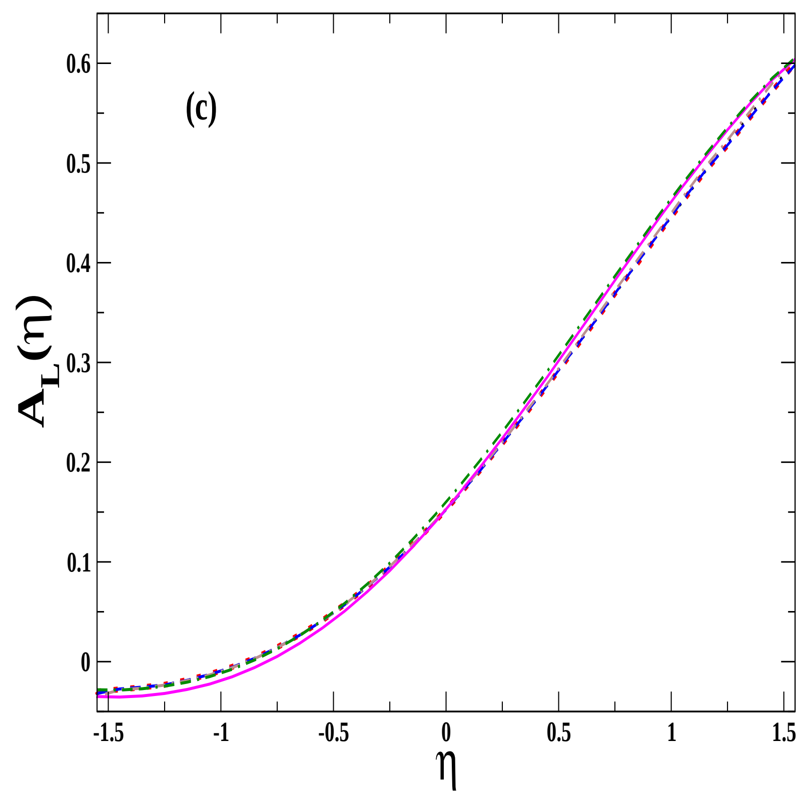


- NNLO terms generated by resummation are still significant, but that orders beyond NNLO have a negligible effect.

- NLO expansion of the resummed cross section reproduces the exact NLO cross section very precisely : logarithmic terms that are subject to resummation indeed dominate the cross section

A_L at RHIC ($\sqrt{S} = 500$ GeV)

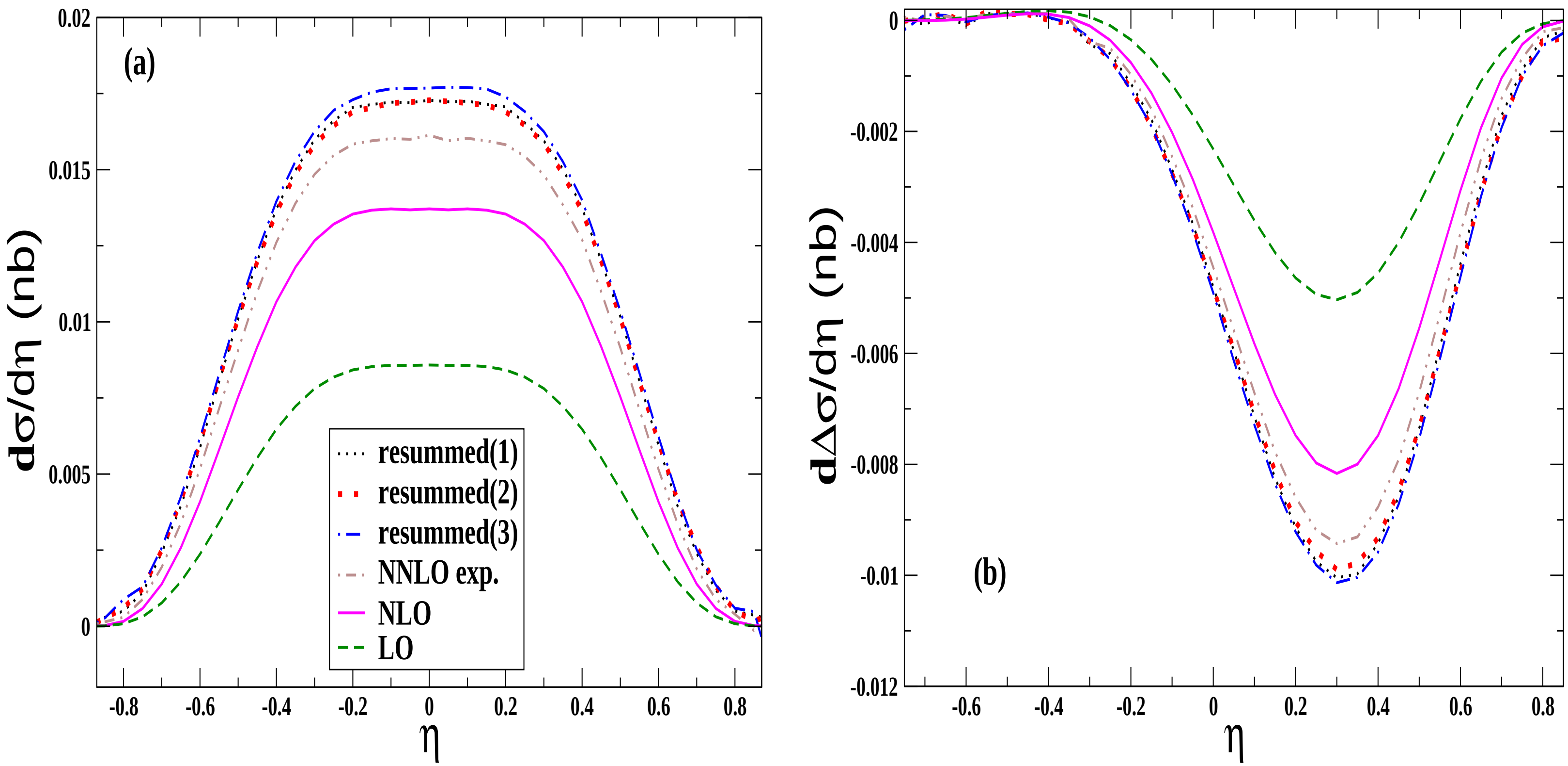




LHS : A_L vs η for W^+ production, RHS : for W^- production

- Cross section : moderate resummation effects at $\sqrt{S} = 500$ GeV
- Cancels in A_L, this property does not depend on pdfs used
- Resummation does not affect the rapidity dependence of the cross section much : shapes of the curves at NLO and NLL similar

W^+ *Production at RHIC* ($\sqrt{S} = 200$ *GeV*)



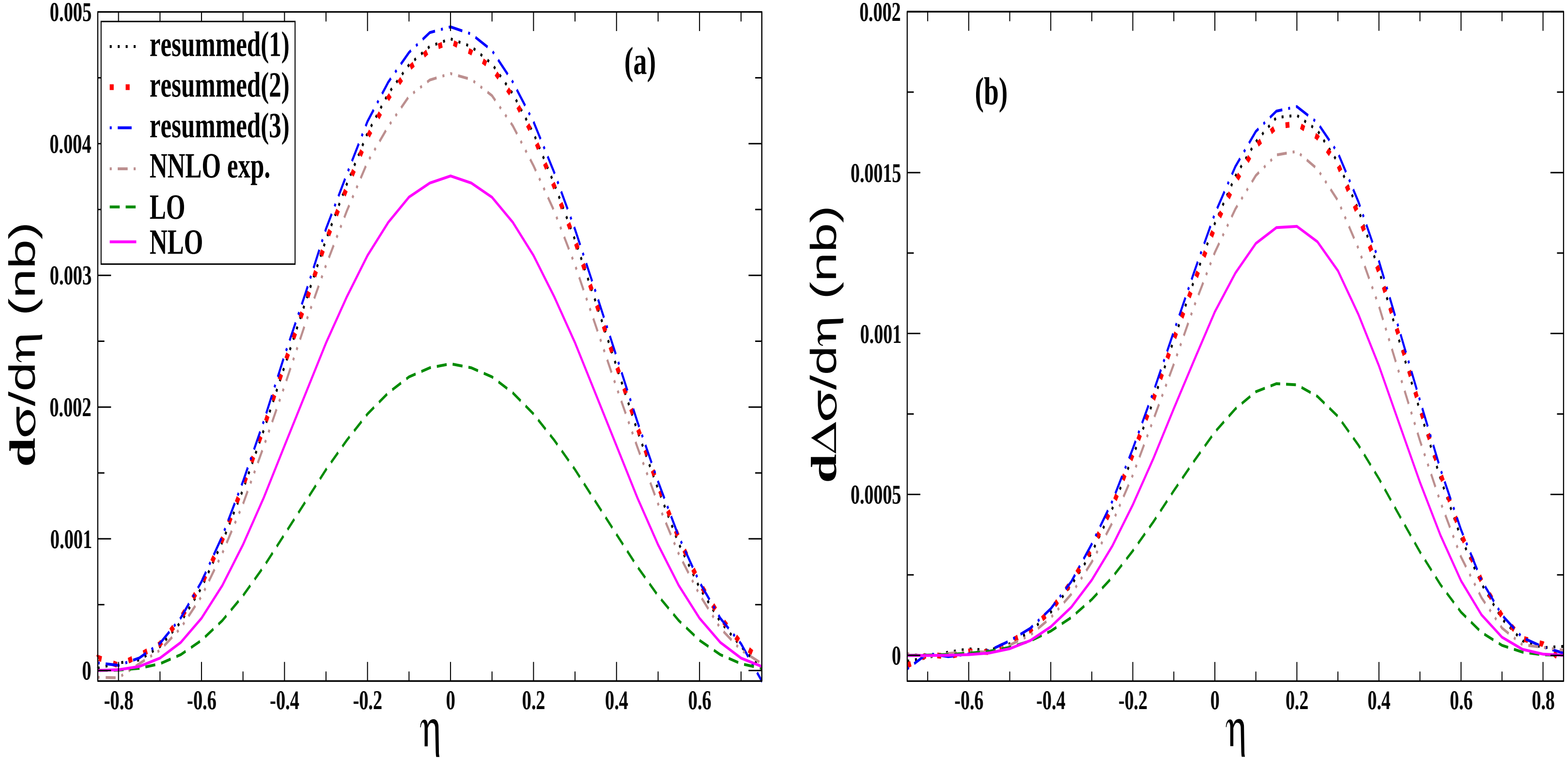
Resummed(1) : threshold-resummed cross section without subleading $\ln(\bar{N})/N$ terms and without the $\cos\left(M \ln \frac{1}{\sqrt{z}}\right)$ term

Resummed(2) : the Cosine term is included

Resummed(3): with $\ln(\bar{N})/N$ terms

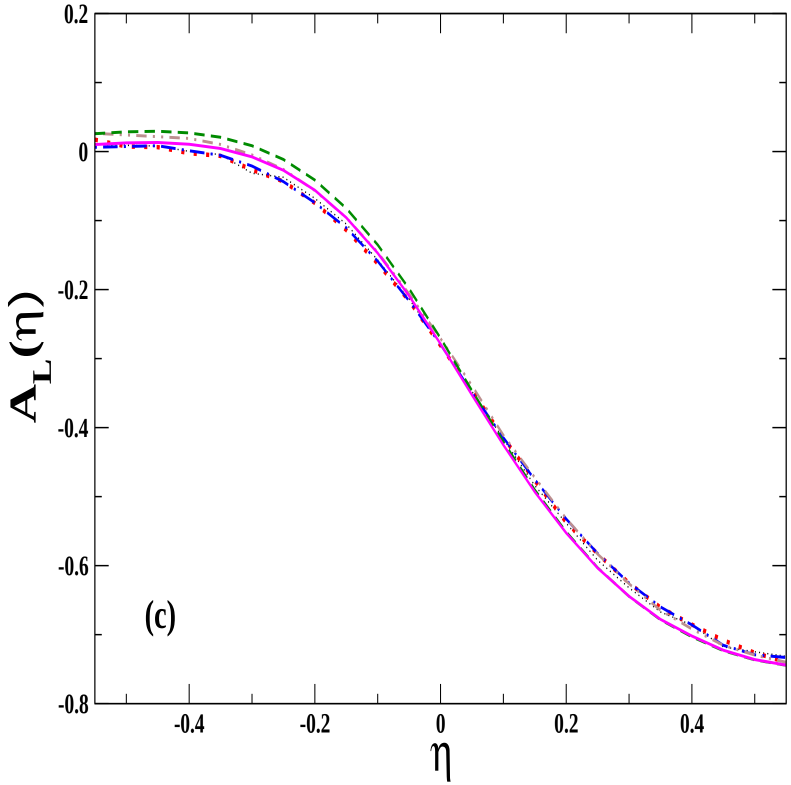
NNLO exp : NNLO expansion of the 'resummed(1)' result

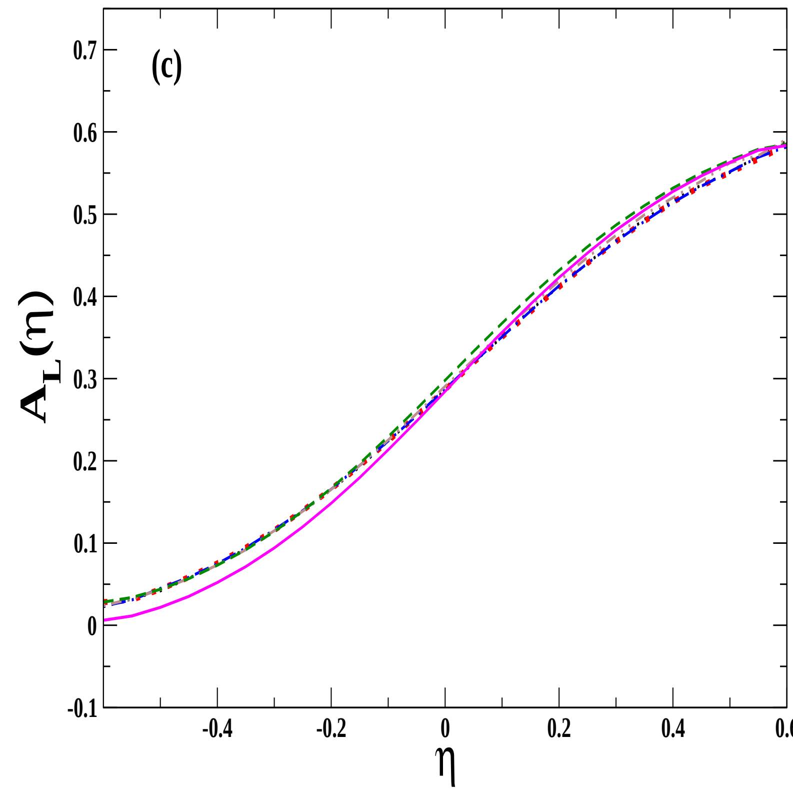
W^- Production at RHIC ($\sqrt{S} = 200$ GeV)



- Larger perturbative corrections expected as closer to threshold
- Resummation effects more significant, 25% at mid-rapidity

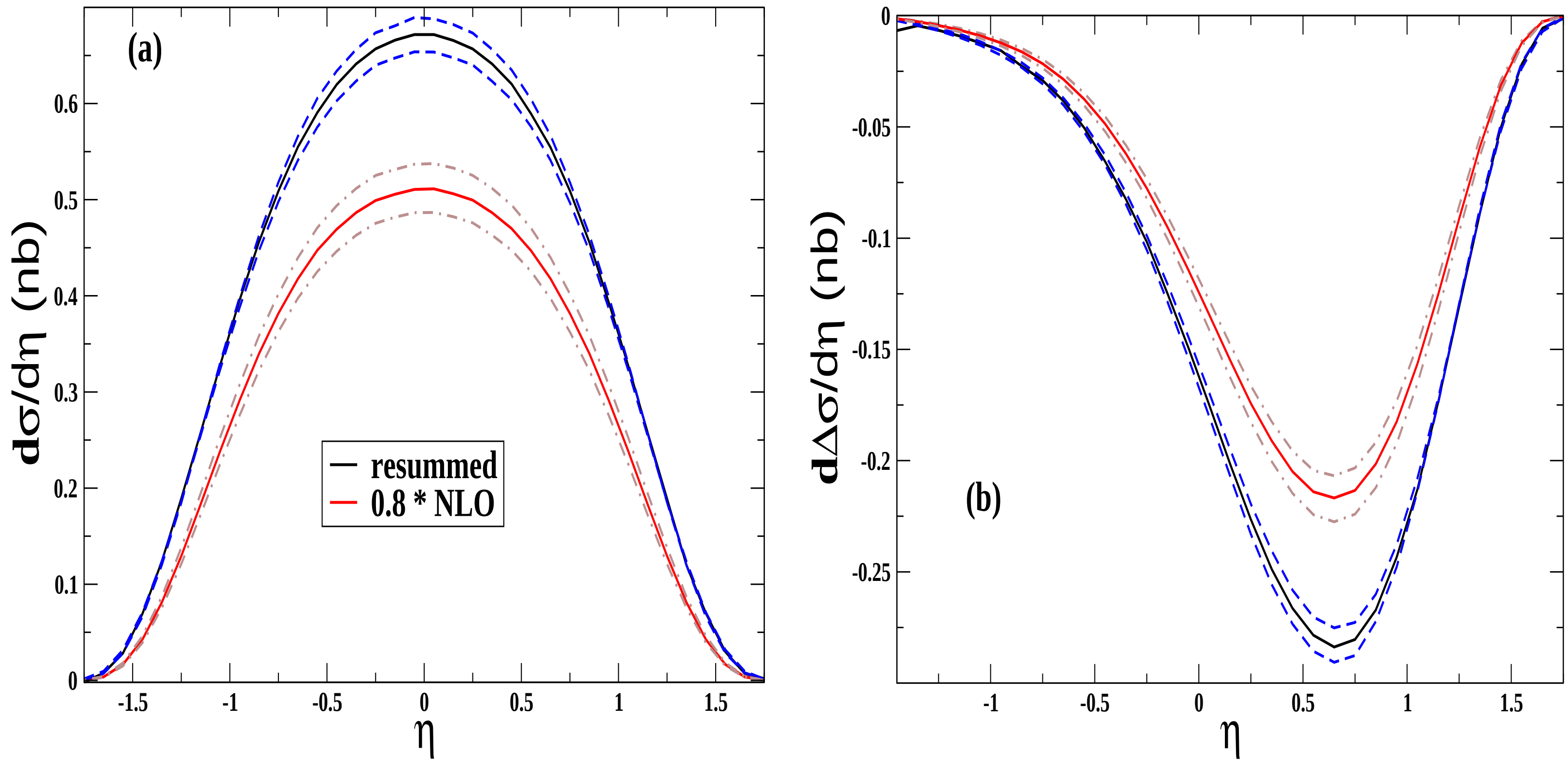
A_L at RHIC ($\sqrt{S} = 200$ GeV)





- LHS : A_L vs η for W^+ production, RHS : for W^- production
- Resummation effects again, cancel almost entirely in A_L

Scale Dependency



Resummed exponent depends on the factorization scales in such a way that it compensates the evolution of the parton distributions : decrease in scale dependence

Scale dependence of the NLO and the ‘resummed(1)” differential cross sections for W^+ production in unpolarized pp collisions at RHIC at $\sqrt{S} = 500$ GeV

Factorization/renormalization scale μ is varied between $M_W/2$ and $2M_W$.

Points to Note

- W observed through leptonic decay $W \to l\nu$; only charged leptons observed at RHIC
- Need to relate lepton kinematics to rapidity of W, and possibly formulate all observables in terms of lepton rapidity
- Interpretation of A_L in terms of the pdfs become more involved, still excellent sensitivity
- Cabbibo suppressed contributions need to be taken into account, also Z boson exchange contributions
- At LO, W has zero p_T. Gluon emission gives recoil p_T to W. As $p_T \to 0$, large logarithms appear in the p_T distribution of $W^{\pm}$. Resummation of these logs has been done in

P. M. Nadolsky and C. P. Yuan, Nucl. Phys. B **666**, 3, 31 (2003);

- Rapidity-differential W cross section : p_T is integrated $\to$ large logarithms turn partly into threshold logarithms and partly into nonlogarithmic terms.
- Recently, threshold resummation of D-Y rapidity distributions has also been discussed in

P. Bolzoni, PLB 643 (2006) 325

Summary and Conclusions

- Performed resummation of potentially large ‘threshold” logarithms that arise when the incoming partons have just sufficient energy to produce the W boson.

- Considered resummation to next-to-leading logarithmic accuracy for the rapidity dependence of the cross sections

- We find that the resummation effect on the unpolarized and single-longitudinally polarized cross sections is rather moderate at RHIC’s higher energy $\sqrt{S} = 500$ GeV, but more significant at $\sqrt{S} = 200$ GeV where one is closer to threshold.

- Shapes of the W rapidity distributions are rather unaffected by resummation.